PARTIALLY QUENCHED CHIRAL PERTURBATION THEORY TO NNLO

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NNLO (Two Loops) in the Chiral Expansion

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  - Meson masses (for $n_f = 2, 3$ flavors of sea quarks)
  - Decay constants (for $n_f = 2, 3$ flavors of sea quarks)
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  - Form factors (possible future extension)
- Application to Lattice QCD simulations with light dynamical sea quarks:
  - Determination of the low-energy constants (LEC:s) of QCD
  - Quark mass depencence of observables, chiral extrapolations

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- Unquenched Lattice QCD simulations (with dynamical sea quarks), are notoriously difficult for physical sea quark masses.
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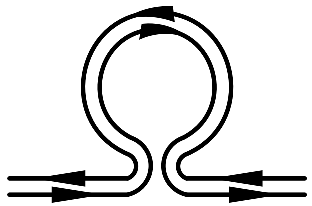
  $\longrightarrow$ No separate mass parameter for the sea quark!

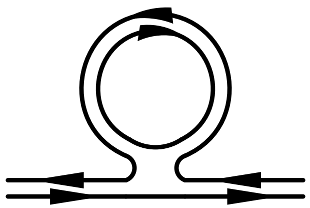
$(\chi\mathrm{PT}) =$

connected



$+$

disconnected



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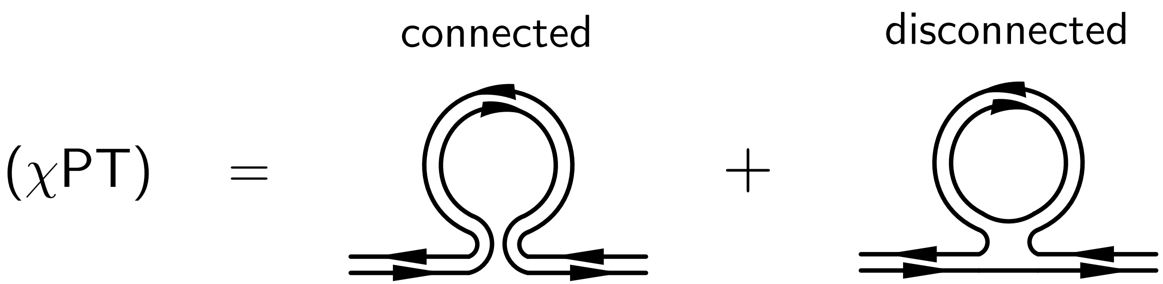


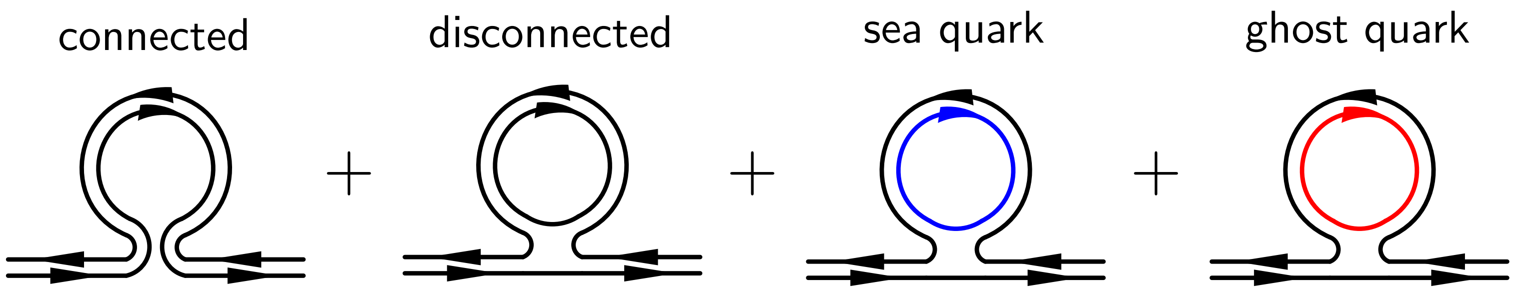
- Conclusion: $\longrightarrow$ Standard χPT is not sufficient for PQ Lattice QCD!

- Generalization of χPT to PQχPT: $\longrightarrow$ Bernard and Golterman, Phys.Rev.**D49**, 468 (1994)

  $\longrightarrow$ add bosonic ghost quark loops to cancel disconnected valence loops

  $\longrightarrow$ add explicit sea quark loops with arbitrary masses

(PQχPT) $=$



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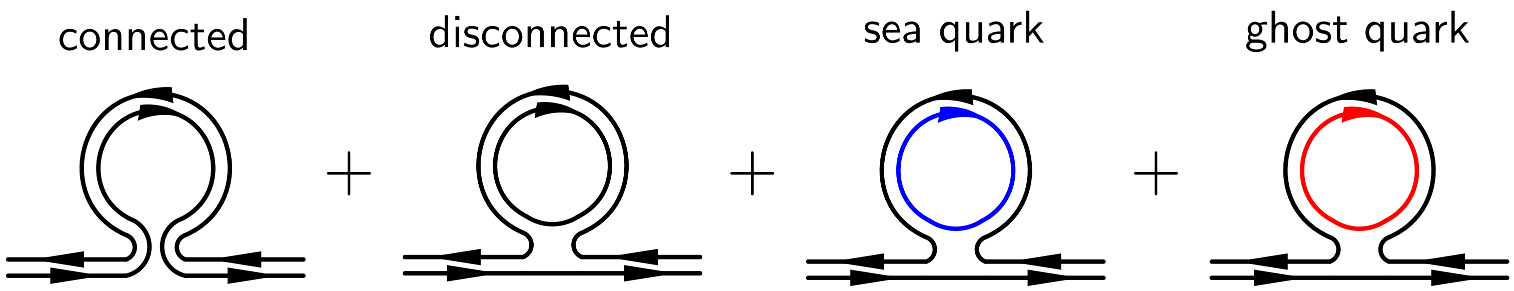
(PQχPT) $=$ 

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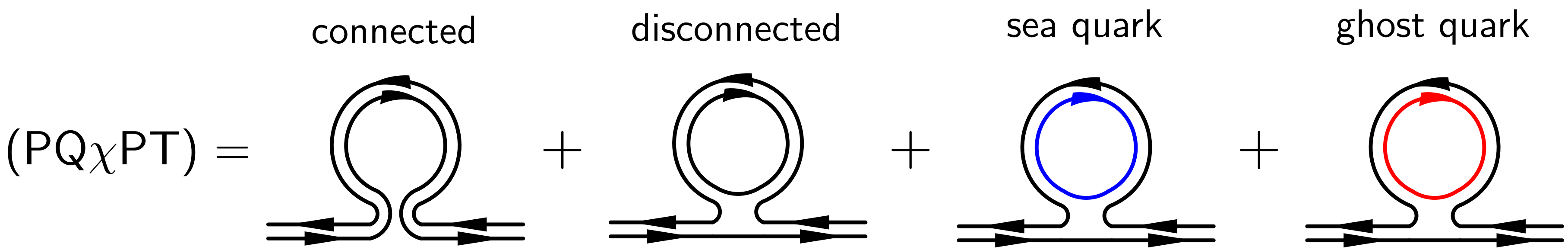

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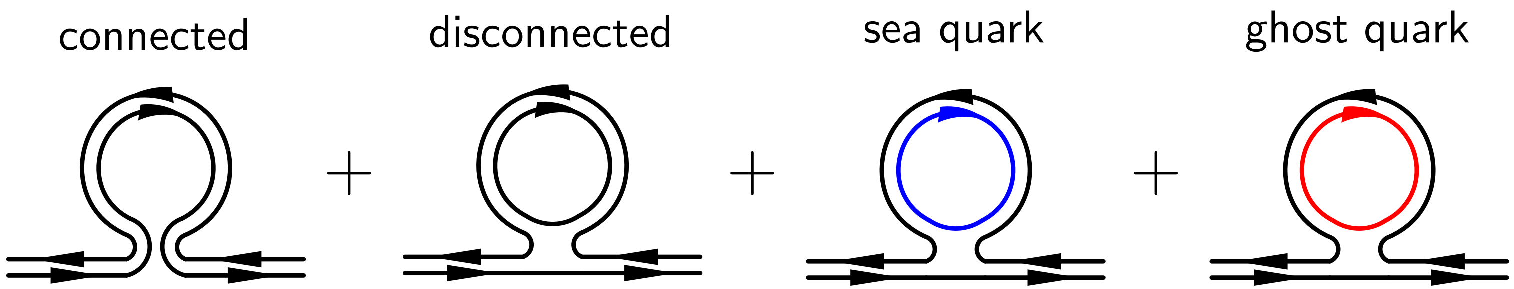
(PQχPT) $=$ 

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- Technical Note III: QCD $\equiv$ PQQCD with equal sea and valence quark masses:
  $\longrightarrow$ LEC:s of QCD can be determined from PQQCD Lattice simulations!

NNLO Calculation of Masses and Decay Constants

- The physical squared masses $M^2_{\rm phys} = M^2$ are calculated from the pole position of the resummed propagator,

$$M^2 = {M_0}^2 + \Sigma(M^2).$$

NNLO Calculation of Masses and Decay Constants

- The physical squared masses $M^2_{\rm phys} = M^2$ are calculated from the pole position of the resummed propagator,

$$M^2 = M_0^2 + \Sigma(M^2).$$

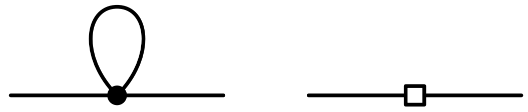
- Up to NNLO, in terms of the lowest order mass M_0 and the self-energy contribution Σ, the masses of the pseudoscalar mesons are given by

$$M^2_{\rm phys} \quad = \quad M_0^2 + \Sigma_4(M_0^2) + \underbrace{\Sigma_4(M_0^2)\,\frac{\partial\Sigma_4(p^2)}{\partial p^2}\bigg|_{M_0^2} + \Sigma_6(M_0^2)}_{\mathcal{O}(p^6)\ \text{contribution}} + \mathcal{O}(p^8).$$

- The self-energy Σ_4 represents the NLO (one-loop) mass shift.

- Feynman diagrams of $\mathcal{O}(p^4)$ which contribute to the NLO meson mass and to the wavefunction renormalization $\sqrt{Z}$: (4 L_i^r:s at NLO)

- $\mathcal{O}(p^2)$ vertices $\longrightarrow$ Filled dots
- $\mathcal{O}(p^4)$ vertices $\longrightarrow$ Open squares



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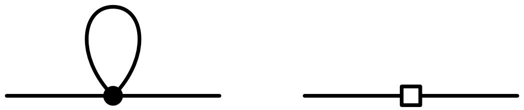
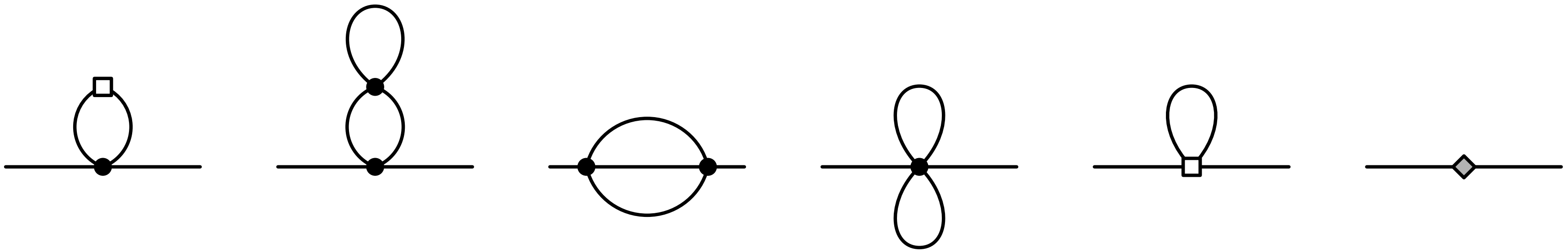


- Feynman diagrams of $\mathcal{O}(p^6)$ which contribute to the NNLO meson mass and to the wavefunction renormalization $\sqrt{Z}$: (9 L_i^r:s and 12 K_i^r:s at NNLO)



- $\mathcal{O}(p^6)$ vertices $\longrightarrow$ Shaded diamonds

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- Up to NNLO, in terms of the lowest order result $F_2 = F_0$ and the self-energy contributions Σ from the wavefunction renormalization, one obtains

$$\begin{aligned} F_{\mathrm{phys}} \;&=\; \underbrace{F_0 + F_4(M_0^2) + F_0\,\frac{\partial\Sigma_4(p^2)}{2\,\partial p^2}\bigg|_{M_0^2}}_{\mathcal{O}(p^4)\ \mathrm{contribution}} + \; F_0\,\frac{3}{8}\left(\frac{\partial\Sigma_4(p^2)}{\partial p^2}\bigg|_{M_0^2}\right)^2 \\ &+\; F_0\,\frac{\partial\Sigma_6(p^2)}{2\,\partial p^2}\bigg|_{M_0^2} + F_4(M_0^2)\,\frac{\partial\Sigma_4(p^2)}{2\,\partial p^2}\bigg|_{M_0^2} + F_6(M_0^2) \;+\; \mathcal{O}(p^8). \end{aligned}$$

- Calculation of the matrix elements F_4 and F_6 requires the evaluation of the contributions from the PQχPT Lagrangians with one external axial current:

- NLO Feynman diagrams for the matrix element F_4: (2 L_i^r:s)

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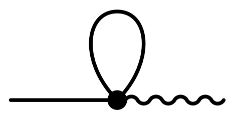

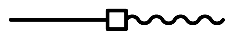

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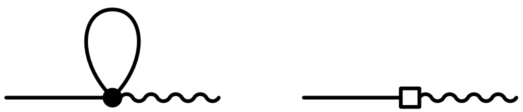

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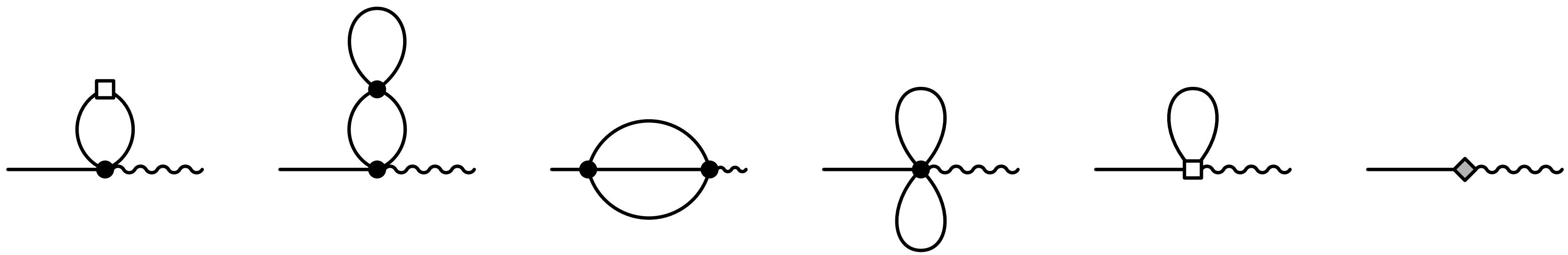

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  $\longrightarrow$ d_{val} distinct valence quark masses,
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- Quark masses through $\chi_i = 2B_0\, m_i$, $\chi_{ij} = (\chi_i + \chi_j)/2$
  $n_f = 2 \longrightarrow$ Valence quarks χ_1, χ_2, sea quarks χ_3, χ_4
  $n_f = 3 \longrightarrow$ Valence quarks χ_1, χ_2, χ_3, sea quarks χ_4, χ_5, χ_6.

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- Loop integrals in PQχPT are more involved than in standard χPT since the PQχPT propagators contain double poles.

- Chiral logarithms $A \sim \chi \log(\chi)$, quenched chiral logarithms $B \sim \log(\chi)$, two-loop sunset integrals H.

- The results for the meson masses and decay constants at $\mathcal{O}(p^4)$ (NLO) and at $\mathcal{O}(p^6)$ (NNLO) are given in terms of the shifts δ_M and δ_F:

$$M^2_{\rm phys} = M_0^2 + \delta_M^{\rm NLO}/F_0^2 + \delta_M^{\rm NNLO}/F_0^4 + \mathcal{O}(p^8)$$

$$F_{\rm phys} = F_0 + \delta_F^{\rm NLO}/F_0 + \delta_F^{\rm NNLO}/F_0^3 + \mathcal{O}(p^8)$$

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- NLO expressions for masses and decay constants:

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$$\begin{aligned}
\delta_M^{\rm NLO} &= -\,24\,L_4^r\,\bar\chi_1\,\chi_{13} - 8\,L_5^r\,\chi_{13}^2 + 48\,L_6^r\,\bar\chi_1\,\chi_{13} + 16\,L_8^r\,\chi_{13}^2 \\
&\quad -\,1/3\,\bar A(\chi_p)\,R^p_{q\pi\eta}\,\chi_{13} - 1/3\,\bar A(\chi_m)\,R^m_{n13}\,\chi_{13} \\
\delta_F^{\rm NLO} &= 12\,L_4^r\,\bar\chi_1 + 4\,L_5^r\,\chi_{13} + \bar A(\chi_p)\,\left[1/6\,R^p_{q\pi\eta} - 1/12\,R^c_p\right] \\
&\quad +\,1/4\,\bar A(\chi_{ps}) - 1/12\,\bar A(\chi_m)\,R^v_{mn13} - 1/12\,\bar B(\chi_p,\chi_p,0)\,R^d_p
\end{aligned}$$

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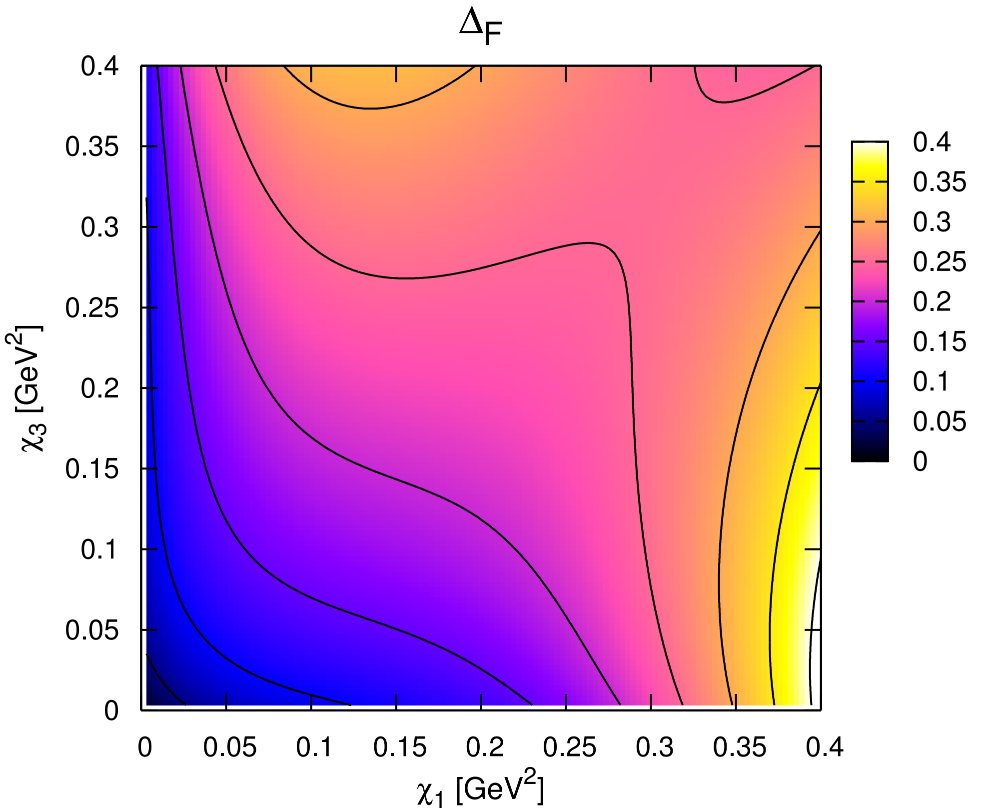
- Example: $n_f = 3$
  NNLO meson mass shift,
  due to chiral logarithms,
  for $d_{\mathrm{val}} = 1$ and $d_{\mathrm{sea}} = 1$
  (about 60 terms): $\longrightarrow$

$$
\begin{aligned}
\delta^{(6)11}_{\mathrm{loops}} = {} & \pi_{16}\, L_0^r \left[3\,\chi_1\chi_4^2 + 26/3\,\chi_1^2\chi_4 - \chi_1^3\right] + 4\,\pi_{16}\, L_1^r\,\chi_1^3 + \pi_{16}\, L_2^r \left[16\,\chi_1\chi_4^2 + 2\,\chi_1^3\right] + \pi_{16}\, L_3^r \left[3/2\,\chi_1\chi_4^2\right. \\
& \left. + 17/3\,\chi_1^2\chi_4 - 5/2\,\chi_1^3\right] + \pi_{16}^2 \left[73/64\,\chi_1\chi_4^2 + 15/32\,\chi_1^2\chi_4 - 3/32\,\chi_1^3\right] + 384\, L_4^r L_5^r\,\chi_1^2\chi_4 - 1152\, L_4^r L_6^r\,\chi_1\chi_4^2 \\
& - 384\, L_4^r L_8^r\,\chi_1^2\chi_4 + 576\, L_4^{r2}\,\chi_1\chi_4^2 - 384\, L_5^r L_6^r\,\chi_1^2\chi_4 - 128\, L_5^r L_8^r\,\chi_1^3 + 64\, L_5^{r2}\,\chi_1^3 - 8\,\bar{A}(\chi_1)\, L_0^r \left[\chi_1^2\right. \\
& \left. + R_1^d\,\chi_1\right] + 8\,\bar{A}(\chi_1)\, L_1^r\,\chi_1^2 + 20\,\bar{A}(\chi_1)\, L_2^r\,\chi_1^2 - 8\,\bar{A}(\chi_1)\, L_3^r \left[\chi_1^2 + R_1^d\,\chi_1\right] + 16\,\bar{A}(\chi_1)\, L_4^r\,\chi_1\chi_4 \\
& + \bar{A}(\chi_1)\, L_5^r \left[32/3\,\chi_1^2 + 16/3\, R_1^d\,\chi_1\right] - \bar{A}(\chi_1)\, L_6^r \left[16\,\chi_1\chi_4 - 32\,\chi_1^2\right] + 32\,\bar{A}(\chi_1)\, L_7^r\, R_1^d\,\chi_1 \\
& - 64/3\,\bar{A}(\chi_1)\, L_8^r\,\chi_1^2 + 5/9\,\bar{A}(\chi_1)^2\,\chi_1 + \bar{A}(\chi_1)\bar{B}(\chi_1,\chi_1,0) \left[11/9\,\chi_1^2 + 1/9\, R_1^d\,\chi_1\right] \\
& + 2/9\,\bar{A}(\chi_1)\bar{C}(\chi_1,\chi_1,\chi_1,0)\, R_1^d\,\chi_1^2 - \bar{A}(\chi_1,\varepsilon)\,\pi_{16} \left[11/12\,\chi_1^2 - 1/4\, R_1^d\,\chi_1\right] + 3\,\bar{A}(\chi_{14})\,\pi_{16}\,\chi_1\chi_4 \\
& + 24\,\bar{A}(\chi_{14})\, L_0^r\,\chi_1\chi_{14} + 60\,\bar{A}(\chi_{14})\, L_3^r\,\chi_1\chi_{14} - 48\,\bar{A}(\chi_{14})\, L_5^r\,\chi_1\chi_{14} + 96\,\bar{A}(\chi_{14})\, L_8^r\,\chi_1\chi_{14} - 9/4\,\bar{A}(\chi_{14})^2\,\chi_1 \\
& - 2\,\bar{A}(\chi_{14})\bar{B}(\chi_1,\chi_1,0)\,\chi_1\chi_4 - \bar{A}(\chi_{14},\varepsilon)\,\pi_{16} \left[9/2\,\chi_1\chi_4 + 5/2\,\chi_1^2\right] + 128\,\bar{A}(\chi_4)\, L_1^r\,\chi_1\chi_4 \\
& + 32\,\bar{A}(\chi_4)\, L_2^r\,\chi_1\chi_4 - 128\,\bar{A}(\chi_4)\, L_4^r\,\chi_1\chi_4 + 128\,\bar{A}(\chi_4)\, L_6^r\,\chi_1\chi_4 + 8/9\,\bar{A}(\chi_4)\bar{B}(\chi_1,\chi_1,0)\,\chi_1\chi_4 \\
& - 2\,\bar{A}(\chi_4,\varepsilon)\,\pi_{16}\,\chi_1\chi_4 - 8\,\bar{B}(\chi_1,\chi_1,0)\, L_0^r\, R_1^d\,\chi_1^2 - 8\,\bar{B}(\chi_1,\chi_1,0)\, L_3^r\, R_1^d\,\chi_1^2 + \bar{B}(\chi_1,\chi_1,0)\, L_4^r \left[8\,\chi_1^2\chi_4\right. \\
& \left. + 24\, R_1^d\,\chi_1\chi_4\right] + \bar{B}(\chi_1,\chi_1,0)\, L_5^r \left[8/3\,\chi_1^3 + 16\, R_1^d\,\chi_1^2\right] - \bar{B}(\chi_1,\chi_1,0)\, L_6^r \left[16\,\chi_1^2\chi_4 + 32\, R_1^d\,\chi_1\chi_4\right] \\
& + 16\,\bar{B}(\chi_1,\chi_1,0)\, L_7^r\,(R_1^d)^2\chi_1 - \bar{B}(\chi_1,\chi_1,0)\, L_8^r \left[16/3\,\chi_1^3 + 32\, R_1^d\,\chi_1^2 - 16/3\,(R_1^d)^2\chi_1\right] \\
& + \bar{B}(\chi_1,\chi_1,0)^2 \left[2/9\, R_1^d\,\chi_1^2 + 1/18\,(R_1^d)^2\chi_1\right] + 2/9\,\bar{B}(\chi_1,\chi_1,0)\bar{C}(\chi_1,\chi_1,\chi_1,0)\,(R_1^d)^2\chi_1^2 \\
& + 29/36\,\bar{B}(\chi_1,\chi_1,0,\varepsilon)\,\pi_{16}\, R_1^d\,\chi_1^2 + 16\,\bar{C}(\chi_1,\chi_1,\chi_1,0)\, L_4^r\, R_1^d\,\chi_1^2\chi_4 + 16/3\,\bar{C}(\chi_1,\chi_1,\chi_1,0)\, L_5^r\, R_1^d\,\chi_1^3 \\
& - 32\,\bar{C}(\chi_1,\chi_1,\chi_1,0)\, L_6^r\, R_1^d\,\chi_1^2\chi_4 - 32/3\,\bar{C}(\chi_1,\chi_1,\chi_1,0)\, L_8^r\, R_1^d\,\chi_1^3 + 5/9\, H^F(1,\chi_1,\chi_1,\chi_1,\chi_1)\,\chi_1^2 \\
& + H^F(1,\chi_1,\chi_{14},\chi_{14},\chi_1) \left[1/4\,\chi_1\chi_4 - \chi_1^2\right] + 2\, H^F(1,\chi_{14},\chi_{14},\chi_4,\chi_1)\,\chi_1\chi_4 \\
& + 4/9\, H^F(2,\chi_1,\chi_1,\chi_1,\chi_1)\, R_1^d\,\chi_1^2 + 3/4\, H^F(2,\chi_1,\chi_{14},\chi_{14},\chi_1)\, R_1^d\,\chi_1^2 + 2/9\, H^F(5,\chi_1,\chi_1,\chi_1,\chi_1)\,(R_1^d)^2\chi_1^2 \\
& - 4\, H_1^F(3,\chi_{14},\chi_1,\chi_{14},\chi_1)\, R_1^d\,\chi_1^2 + 3/4\, H_{21}^F(1,\chi_1,\chi_{14},\chi_{14},\chi_1)\,\chi_1^2 + 6\, H_{21}^F(1,\chi_4,\chi_{14},\chi_{14},\chi_1)\,\chi_1^2 \\
& - 3/4\, H_{21}^F(2,\chi_1,\chi_{14},\chi_{14},\chi_1)\, R_1^d\,\chi_1^2.
\end{aligned}
$$

Decay Constant Shift Δ_F for $n_f = 2$ with $d_{\mathrm{val}} = 1, d_{\mathrm{sea}} = 1$

- Δ_F for NLO + NNLO $\longrightarrow$
- Relative shift Δ_F:
  $F_{\mathrm{Phys}} = F_0\,(1 + \Delta_F)$
- L_i^r from χPT "fit 10" $\longrightarrow$
  Amorós *et al.*, Nucl.Phys.**B602**, 87 (2001)
- K_i^r all set to zero $\longrightarrow$
- $\chi_i \sim 0.01$ GeV^2 for a 100 MeV physical meson mass
- Validity range of PQχPT:
  All $\chi_i \leq 0.3$ GeV^2.



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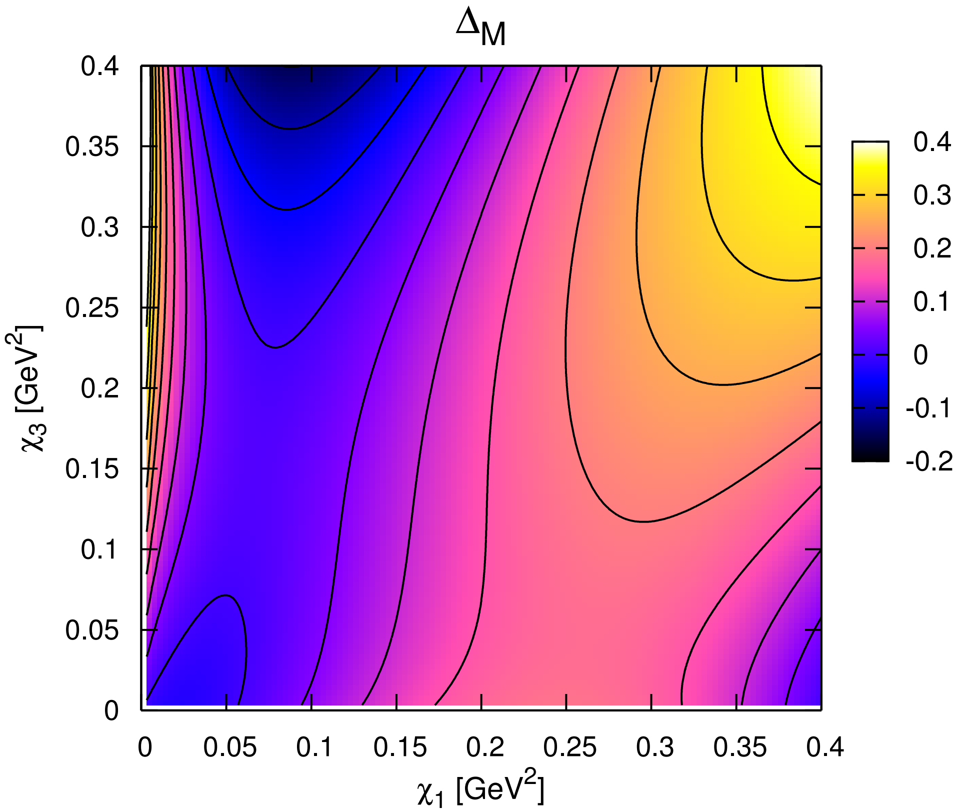
- Validity range of PQχPT:
  All $\chi_i \leq 0.3$ GeV^2.



Published NNLO calculations in PQχPT:

- Phys.Rev.**D70**:111503 (2004), hep-lat/0406017,
  Pseudoscalar meson mass to two loops in three-flavor partially quenched chiral perturbation theory.
  $\longrightarrow$ Introductory letter on PQχPT at NNLO
- Phys.Rev.**D71**:094502 (2005), hep-lat/0501014,
  Decay constants of pseudoscalar mesons to two loops in three-flavor partially quenched chiral perturbation theory.
  $\longrightarrow$ Pseudoscalar meson decay constants for $n_f = 3$
- Phys.Rev.**D72**:074502 (2005), hep-lat/0506004,
  Masses and decay constants of pseudoscalar mesons to two loops in two-flavor partially quenched chiral perturbation theory.
  $\longrightarrow$ Pseudoscalar meson masses and decay constants for $n_f = 2$
- In preparation, to be submitted to Phys.Rev.**D**,
  $\longrightarrow$ Pseudoscalar meson masses for $n_f = 3$

PQχPT results available on the net:

- Complete masses and decay constants to NNLO for $n_f = 2$
  $\longrightarrow$ http://www.thep.lu.se/∼bijnens/chpt.html
- NNLO results for $n_f = 3$ will appear soon

To appear in the future:

- Further analysis and fitting strategies
- PQ Masses and decay constants at Finite Volume
  $\longrightarrow$ Work in progress (T.L, Johan Bijnens, Karim Ghorbani)
- PQ Electromagnetic form factors